Discrete homoclinic orbits in a laser with feedback

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We provide experimental evidence of the discrete character of homoclinic chaos in a laser with feedback. We show that the narrow chaotic windows are distributed exponentially as a function of a control parameter. The number of consecutive chaotic regions corresponds to the number of loops around the saddle focus responsible for Shilnikov chaos. The characterization of homoclinic chaos is also done through the return map of the return times at a suitable reference point.

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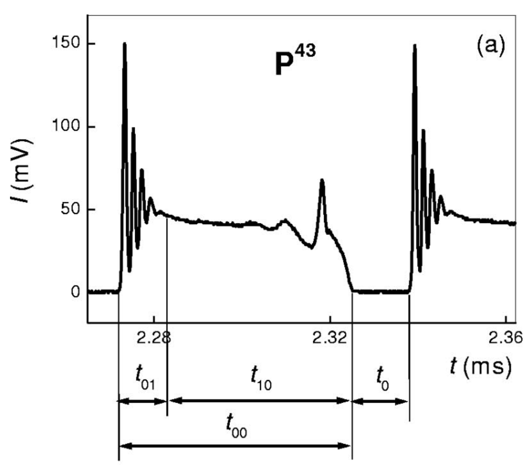
Experimentally, Shilnikov chaos [1] has been previously observed in a single mode laser with feedback [2], in the Belousov-Zhabotinskii reaction [3], in a laser with a saturable absorber [4], in a glow discharge plasma [5], in an optically bistable device [6], in a multimode laser [7], and in some other systems. The detailed aspects of the homoclinic chaos depend sensitively on the setting of the control parameters. Normally homoclinic orbits are located within very narrow parameter ranges between periodical ranges. Therefore, in experiments it is difficult to fix the system in the neighborhood of a homoclinic orbit because of noise or small unavoidable fluctuations of the system parameters. In the present and earlier experiments on a single-mode CO_2 laser with feedback [2], a careful setting of the parameters provides orbits characterized by a large damped oscillation with n periods followed by a smaller growing oscillation with m periods.

The example of the experimental time series corresponding to the P^{43} periodic orbit is shown in Fig. 1(a). This regime is characterized by the alternation of four short and three long oscillations and the corresponding duration times have been called, respectively, t_{01} and t_{10}. In the corresponding phase-space diagram presented in Fig. 1(b), one can see $n=4$ inward and $m=3$ outward spirals around the saddle point 1. The overall time associated with the two spirals is denoted as t_{00}. After a time interval t_0 spent near point 0, another homoclinic cycle starts. By varying the control parameters, one can change the number of the loops, or, in other words, the type of periodicity. Within very narrow parameter ranges, the inward spiral transforms to the quasihomoclinic orbit associated with the outward spiraling saddle set, i.e., with the fixed point 1. The number of outward spiral loops m depends on how close the trajectory approaches fixed point 1 after the reinjection. Therefore, the time t_{10} varies from one homoclinic cycle to the another. Theoretically, the number of quasihomoclinic cycles increases without a limit as the system approaches the homoclinic bifurcation. When the control parameter is changed, m changes discretely by adding or deleting one loop. Each change in m is accompanied by a different chaotic motion. Thus the homoclinic chaos has a discrete behavior with respect to the control parameters.

The present Brief Report is primarily concerned with



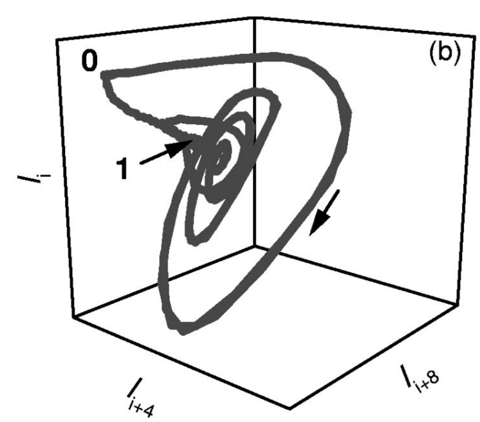


FIG. 1. (a) Experimental time series of the laser intensity and definitions of the characteristic times t_{01}, t_{10}, t_{00}, and t_0. (b) Corresponding phase-space trajectory on the 3D projection of the 6D space. The phase space is built by an embedding technique with appropriate delays. Bias voltage $B=210$ V. Feedback gain $A=2055$.

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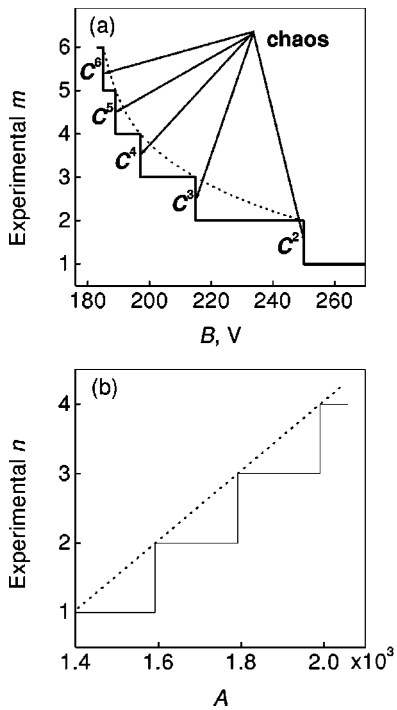

FIG. 2. Discretization of dynamical states. Experimental numbers (a) m and (b) n vs feedback gain. The fixed control parameters are (a) $A=1730$, (b) $B=210$ V.

identifying regions of operation in which homoclinic chaos can be observed, even though such regions can be quite narrow. We demonstrate experimentally the discrete behavior of this homoclinic chaos, that is, we reveal how by changing a control parameter one can change the number m of loops around the saddle focus. To our knowledge, no accurate characterization of the discrete behavior of homoclinic chaos has been previously reported in physical systems, although the effect of discretization has been described in the mathematical literature [8]. We show how these regions depend on the control parameters. We also characterize homoclinic chaos by the return map of the return times (RMRT), as introduced in Ref. [9] and later applied in Ref. [10].

The experiments have been carried on a single-mode CO_2 laser with feedback. The experimental setup was already described elsewhere [11]. An intracavity loss modulator is driven by a bias voltage B, which acts as a control parameter, plus a signal feedback from a detector inspecting the amount of output intensity, and amplified by a factor A. A is the second control parameter. Thus, the laser with feedback represents an autonomous system, where the feedback provides an additional degree of freedom, which is necessary to observe chaos. In Fig. 2, we show how the number of loops depends on the control parameters. We have found that m is separated *exponentially* with the bias voltage, while n changes approximately equidistantly with the feedback gain. Each steep change of m is accompanied by the very narrow chaotic regions C^j indicated by the arrows, where j is the number of a chaotic state that corresponds to the number of the loops m around fixed point 1 [Fig. 2(a)]. Instead, the number of loops in the inward spiral is not associated with chaos and changes equidistantly with a parameter [Fig. 2(b)]. We have detected up to six homoclinic chaotic states, since the maximum observed loops m was six.

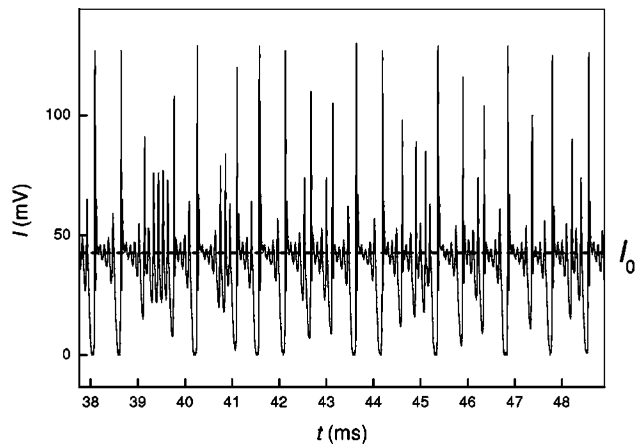

FIG. 3. Portion of time series for chaotic window C^5. For calculating RMRT, the signal is sampled at $I_0=44$ mV.

Return time maps are an extremely convenient tool for analyzing homoclinic chaos [9,10]. Rather than looking at the reconstructed phase-space trajectory, the information is derived directly from the time evolution of the signal. We recall (see Refs. [9,11]) that a map of the global return times can be built by the local analysis around the saddle focus. A map of the global return time in the chaotic regime is made of many oscillations crossing the diagonal straight line of the plane (t_{n+1},t_n) (locus of fixed points) at angles much larger than 45°, thus showing very high local expansion rates. As a result, tiny changes in a control parameter may induce dramatic changes in the range of return times. A return time return map, i.e., the plot t_{i+1} versus t_i, can be derived from the temporal evolution of the laser intensity (Fig. 3). We consider the intersections with the surface of constant intensity ($I=44$ mV) that cuts all oscillations whenever $dI/dt<0$ [10], then we determine the times t_i between successive intersections. The experimental RMRT corresponding to chaos C^5 is shown in Fig. 4. One can see that the RMRTs present a multivalued structure. This map is quite different from those reported in Ref. [9], which were taken setting a high threshold value, and thus reporting the occurrence of the highest spikes. In Ref. [9] no detail on the number of loops around saddle focus 1 could be obtained. This weak point of the previous investigation is amended by the experiment of the present work.

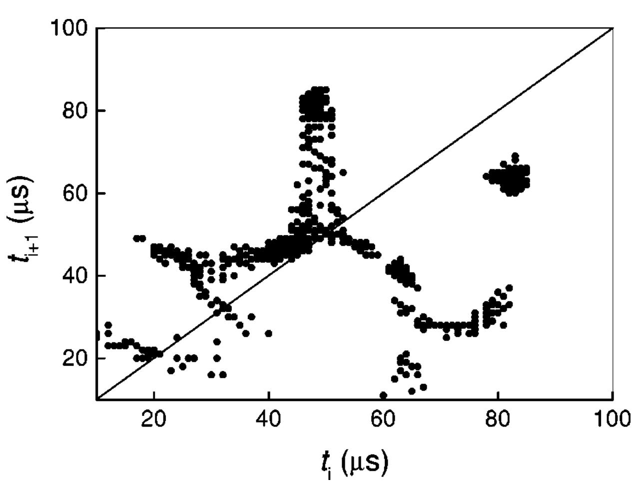

FIG. 4. Experimental RMRT for C^5.

Thus, the performed time series and return time analysis of the experimental data confirms the discrete character of homoclinic chaos. This discrete character had already been observed in the Belousov-Zhabotinskii reaction [3], however the exponential dependence of periodic and chaotic windows is demonstrated here. In conclusion, while previous works monitored the overall return time [9] or stabilized the fixed point 1 [12] or just the first limit cycle around it [13], here we provide the parameter value for the onset of a different number of loops around the saddle focus.

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